Section 6.6

Math 231

Hope College

Newton's Law of Cooling

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference in temperature between the object and its surroundings. If $y(t)$ is the object's temperature at time t and S is the temperature of the surroundings (assumed to be constant), this results in the first-order autonomous IVP

$$y' = k(S - y), \qquad y(0) = y_0,$$

where y_0 is the temperature of the object at time $t = 0$ and k is a constant.

Suppose a cup of coffee, originally at 92°C is placed in a room at noon where the temperature is 22°C. At 12:10pm, the temperature of the coffee is measured to be 85°C. Use your answer to (b) to predict the temperature of the coffee at 12:30pm.

When is the temperature of the coffee equal to 70°C.

Example of a Mixing Problem

Suppose a tank is filled with 10L of solution. At time $t = 0$, you begin adding solution with a salt concentration of 30 g/L to the tank at a rate of 2 L/minute. Well-mixed solution is allowed to leave the tank at the same rate. Find a formula for $Q(t)$, the number of grams of salt in the tank at time t minutes. Also, find the limiting concentration of solution in the tank as $t \to \infty$.

For part (a), assume that the tank initially has pure water in it. For part (b), assume that the 10L of solution in the tank has an initial salt concentration of 50 g/L.